

A NOTE ON AN EQUIVALENT OF THE RIEMANN HYPOTHESIS

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Abstract: In this manuscript we denote by \sum_{ρ} a sum over the non trivial zeros of Riemann zeta function (or over the zeros of Riemann's xi function), where the zeros of multiplicity k are counted k times. We prove a result that the Riemann Hypothesis is true if and only if

$$\sum_{\rho} \frac{1}{|\frac{1}{2} - \rho|^4} = \frac{1}{2} \left(\frac{\xi''(\frac{1}{2})}{\xi(\frac{1}{2})} \right)^2 - \frac{1}{6} \left(\frac{\xi^{(4)}(\frac{1}{2})}{\xi(\frac{1}{2})} \right)$$

Keywords and Phrases: Riemann zeta function, Riemann xi function, Riemann Hypothesis, Hadamard product.

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1. Introduction and Definitions

The Riemann zeta function, $\zeta(s)$ is defined as the analytic continuation of the Dirichlet series

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$