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FRACTIONAL CALCULUS BETWEEN BANACH SPACES ALONG WITH OSTROWSKI AND GRÜSS TYPE INEQUALITIES

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Abstract: Here we present a fractional calculus Caputo type for functions between Banach spaces. Left and right fractional derivatives are defined on a segment of a Banach space, then we expand. We apply the above to new Ostrowski and Grüss type inequalities at very abstract level.

Keywords and Phrases: Abstract fractional calculus, abstract fractional derivative, Ostrowski inequality, Grüss inequality.

2020 Mathematics Subject Classification: 26A33, 26D10, 26D15.

1. Introduction

The problem of estimating the difference of a value of a function from its average is a top one. The answer to it are the Ostrowski type inequalities. Ostrowski type inequalities are very useful among others in Numerical Analysis for approximating integrals. The problem of estimating the difference between the average of a product of functions from the product of their averages is also a very important one. The answer to it are the Grüss type inequalities. Grüss type inequalities are very useful among others in Probability for estimating expected values, etc. There exists a huge literature on Ostrowski and Grüss type inequalities to all possible directions. Mathematical community is very much interested to these inequalities due to their applications. So here we derive very general fractional Ostrowski and Grüss type inequalities on a very abstract level. Our functions are between Banach