

## ITERATION OF $n$ ENTIRE FUNCTIONS WITH FINITE ITERATED ORDER

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**Abstract:** After the works of Banerjee and Adhikary [1] on composition of three entire functions with finite iterated order in this paper we investigate some growth properties of  $n$  iterated entire functions of finite iterated order.

**Keywords and Phrases:** Order, Iterated  $i$ -order, Entire function, Composition.  
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### 1. Introduction and Definitions

If  $f(z)$  and  $g(z)$  be two transcendental entire functions, Clunie [5] showed that  $\lim_{r \rightarrow \infty} \frac{T_{f \circ g}(r)}{T_f(r)} = \infty$  and  $\lim_{r \rightarrow \infty} \frac{T_{f \circ g}(r)}{T_g(r)} = \infty$ . After this many authors [3, 4, 6, 7, 8, 9, 10, 12] made close investigation on composition of two entire functions with finite order and obtained various results. Recently Jin Tu et.al [11] investigated the composition of entire functions with finite iterated order and proved results on comparative growths of  $\log^{[p+q]} T_{f \circ g}(r)$  ( $p, q \in \mathbb{N}$ ) with  $\log^{[p]} T_f(r)$  and  $\log^{[q]} T_g(r)$ . In this paper we study some properties on iteration of functions with finite iterated order and extend some earlier results of Banerjee and Adhikary [1] for composition of  $n$  entire functions. We first recall the notion of iterated order [7].

**Definition 1.1.** The iterated  $i$  order  $\rho_i(f)$  and iterated  $i$  lower order  $\mu_i(f)$  of an entire function  $f$  are defined by

$$\rho_i(f) = \limsup_{r \rightarrow \infty} \frac{\log^{[i+1]} M_f(r)}{\log r} = \limsup_{r \rightarrow \infty} \frac{\log^{[i]} T_f(r)}{\log r}, \quad (i \in \mathbb{N})$$