

Some Problems Connected with Ramanujan's Hypergeometric Series

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Dedicated to Prof. Hari M. Srivastava on his 75th birth anniversary

Abstract: Starting from Gauss hypergeometric function, which Ramanujan studied in detail, limiting forms are considered and pathways are constructed to go from a Gauss hypergeometric series to confluent hypergeometric series to binomial series to Bessel series and finally to exponential series. The path from binomial to exponential is discussed in detail. It is shown that this path is connected to reaction-rate probability integrals, non-extensive statistical mechanics, Tsallis statistics, superstatistics, Krátzel integrals, inverse Gaussian density Bayesian procedures and generalizations of these. Products of binomials forms are shown to be connected to Mellin convolutions, densities of products and ratios and finally to fractional integrals. Some aspects of the corresponding theory of functions of matrix argument are also discussed.

Keywords Hypergeometric functions, pathway models, reaction-rate probability integrals, Krátzel integrals, fractional integrals, Mellin convolutions, functions of matrix argument, densities of product and ratios.

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1. Some Classical Limiting Properties of Hypergeometric Series

One of the areas of interest for Ramanujan was Gauss hypergeometric series ${}_2F_1(a, b; c; x)$. By specializing the parameters here he obtained several interesting relationships. Several people are still working in this area of specializing parameters and constructing results. One classical property of Gauss hypergeometric series is the following:

$$\lim_{a \rightarrow \infty} {}_2F_1\left(a, b; c; \frac{x}{a}\right) = {}_1F_1(b; c; x) \quad (1.1)$$