J. of Ramanujan Society of Math. and Math. Sc. Vol.2, No.1 (2013), pp. 109-128

A note on Bailey pairs and q-series identities

ISSN: 2319-1023

(Received April 05, 2013)

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Abstract: In this paper, making use of Barley pairs and a general transformation formula we have established interesting q-series identities.

Keywords and Phrases: Bailey Pair, Transformation formula, Summation formula and Identity.

2000 AMS Subject Classifications: 33D15

1. Introduction

As usual, we employ the notations,

$$(a;q)_n = (1-a)(1-aq)...(1-aq^{n-1}), \quad n \ge 1,$$

 $(a;q)_0 = 1, \quad (a;q)_\infty = \prod_{n=0}^\infty (1-aq^n)$

and

$$(a_1, a_2, ..., a_r; q)_n = (a - 1; q)_n (a_2; q)_n ... (a_r; q)_n$$

An $_r\Phi_s$ basic hypergeometric series

$$_{r}\Phi_{s}\left[\begin{array}{c}a_{1},a_{2},...,a_{r};q;z\\b_{1},b_{2},...,b_{s}\end{array}\right]=\sum_{n=0}^{\infty}\frac{[a_{1},a_{2},...,a_{r};q]_{n}z^{n}}{[q,b_{1},b_{2},...,b_{s};q]_{n}},\quad|z|<1$$

Bailey is 1947 showed that

If
$$\beta_n = \sum_{r=0}^n \frac{\alpha_r}{[q;q]_{n-r}[aq;q]_{n+r}}$$
 and $\gamma_n = \sum_{r=n}^\infty \frac{\delta_r}{[q;q]_{r-n}[aq;q]_{r+n}}$ then
$$\sum_{n=0}^\infty \alpha_n \gamma_n = \sum_{n=0}^\infty \beta_n \delta_n. \tag{1.1}$$